

# Simulation and analysis of microwave transmission through an electron cloud, a comparison of results

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## Abstract

Simulation studies for transmission of microwaves through electron cloudes show good agreement with analytic results. The electron cloud produces a shift in phase of the microwave. Experimental observation of this phenomena would lead to a useful diagnostic tool for accessing the local density of electron clouds in an accelerator. These experiments are being carried out at the CERN SPS and the PEP-II LER at SLAC and is proposed to be done at the Fermilab maininjector. In this study, a brief analysis of the phase shift is provided and the results are compared with that obtained from simulations.

## INTRODUCTION

This paper shows some preliminary results of work in progress for a planned exhaustive study on the physical processes involved with the transmission of microwaves through a beam channel containing electron clouds. Successful experimental observation of the phase properties of the microwave after the transmission would serve as a very useful probe for detecting electron clouds and their properties at different locations of a beam channel. Given the complexity of the problem, and in order to be able to correlate the properties of the electron distribution with the expected experimental observations, a good understanding of the problem through simulations and theoretical analysis is necessary.

Experimental results conducted at the SPS have been reported [1], and some preliminary experimental investigation has also been conducted at the PEP II low energy ring [4] will be reported. While it is clear that more progress is required in the experimental and simulation efforts, we can already state that results obtained so far are encouraging. Further, simulation results presented here, which are done for idealized conditions show good agreement with analytic results.

To simulate the physical process, we use the electromagnetic particle-in-cell code VORPAL [2]. For the sake of simplicity, and as an initial part of the study, we use a channel with a square cross-section. The transverse boundaries are conducting and boundaries along the longitudinal direction consist of phase matched layers (PML) which cause the wave to decay and be absorbed, thus preventing any reflection in that direction. Figure 1 shows a snapshot of the simulation process.

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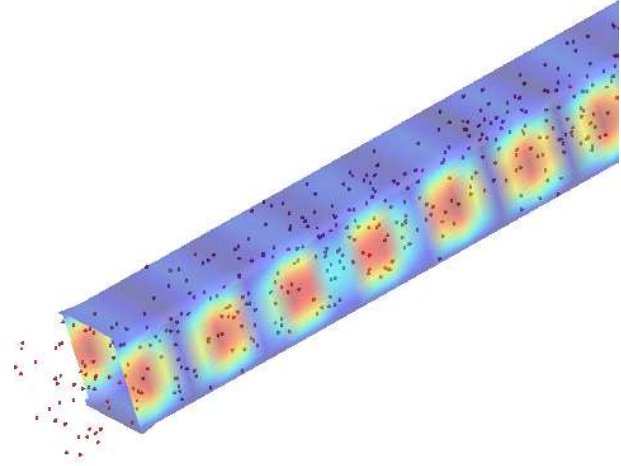


Figure 1: A schematic of the simulation process. The microwave source is placed a small length away from the left boundary

## DISCUSSION OF THE DISPERSION RELATION AND THE RESULTING PHASE SHIFT

The method for deriving the wave dispersion relationship has been described in Ref. [3]. The derivation involves a perturbation about an equilibrium configuration of the Maxwell and fluid equations and where all higher order perturbation terms are neglected. As a result, the electron equation of motion is linear. In this model we do not expect any attenuation in the amplitude of the wave and the frequency spectrum remains invariant.

For a cold, homogeneous electron distribution that completely fills the beam channel and with no external magnetic field, the wave dispersion relationship for a TE mode is relatively simple and given by

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} - \frac{\omega_c^2}{c^2} \quad (1)$$

Over here,  $k$  is the wave number,  $c$  the speed of light,  $\omega$  is the wave frequency,  $\omega_p$  is the plasma frequency and  $\omega_c$  is the cutoff frequency of the waveguide. The plasma fre-

quency  $\omega_p$  is given by

$$\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad (2)$$

where  $n_e$  is the electron number density,  $e$  is the electron charge,  $\epsilon_0$  is the free space permittivity and  $m_e$  is the electron mass.

If we assume the wave to propagate over a distance  $\Delta L$ , then the phase advance of the wave is given by  $k\Delta L$ . It is clear from Eq. (1) that the phase advance of the wave depends upon the plasma frequency, a property of the electron distribution. Thus, if we compute the difference between the phase advance with and without electrons respectively, we get

$$\Delta\Phi = [(\omega^2 - \omega_c^2)^{1/2} - (\omega^2 - \omega_c^2 - \omega_p^2)^{1/2}] \frac{\Delta L}{c} \quad (3)$$

If we linearize this about small  $\omega_p$  we obtain for a unit length

$$\Delta\phi = \frac{\omega_p^2}{2c(\omega^2 - \omega_c^2)^{1/2}} \quad (4)$$

This is the so called “phase shift” per unit length.

Figure 2 shows plots of the phase shift against frequency based on Eq (4) for different electron densities. The electron densities used here are  $1 \times 10^{12}$ ,  $2 \times 10^{12}$ ,  $4 \times 10^{12}$  and  $7 \times 10^{12}$  and the corresponding plasma frequencies are 8.9, 12.7, 18.0 and 23.8 MHz respectively. The cutoff frequency of the waveguide 2GHz. It should be noted that this is valid for small  $\omega_p$  in Eq (1), so one must take the exact relationship when  $\omega$  approaches to a value very close to  $\omega_c$ . The plots give an indication of what an optimum value of the microwave frequency should be in order to determine the electron density. Higher frequencies lead to a lower variation in phase shift with density. Lowering the frequency closer to the cutoff is not always possible in order for the signal to be detectable. In this case, we expect that a microwave frequency of 3GHz is reasonable.

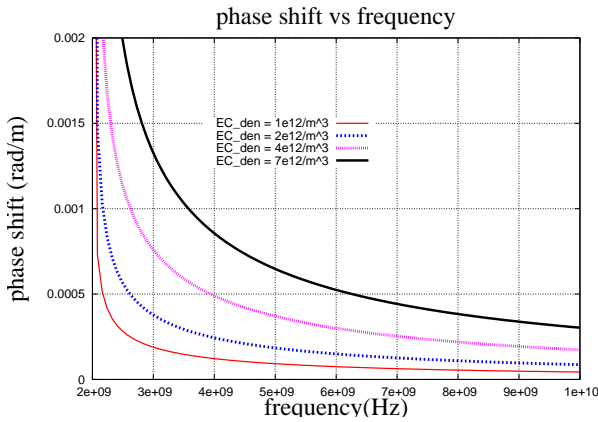


Figure 2: Plots of phase shift per unit length for different electron densities with  $\omega_c = 2\text{GHz}$

The analytic results in this section provide a good understanding of the problem, but one must take into account

that they are valid under certain idealized conditions. For example, the formulation above assumes a cold, homogeneous electron distribution with no external magnetic field. Further, the electron equation of motion is linearized in a quasi-equilibrium regime. In reality, there is a spatial and temporal variation in the electron density. One must also take into account the periodic passage of the beam, which provides a time varying electric field. In particle-in-cell simulation, these approximations may be relaxed. In the next section we shall show through simulations that real accelerator conditions fall into a regime where many of the above assumptions hold, and so the analytic result is a good starting point in understanding the problem.

## SIMULATION RESULTS AND CALCULATION OF PHASE SHIFT

In this section, we present the simulation results obtained from the electromagnetic particle-in-cell code VORPAL. The system consists of a channel with a square cross section. The electron distribution fills up the whole channel and is initially uniform and cold. Other parameters used in the simulation are given below.

Table 1: Simulation parameters

dimensions	7.5 cm (x,y)	0.5 m (z)
no. of grid cells	16 (x,y)	32 (z)
time of propagation	$7.0 \times 10^{-8}$ sec	
number of time steps	7560	
corresponding cutoff frequency for $TE_{01}$	2 GHz	
microwave frequency	3 GHz	

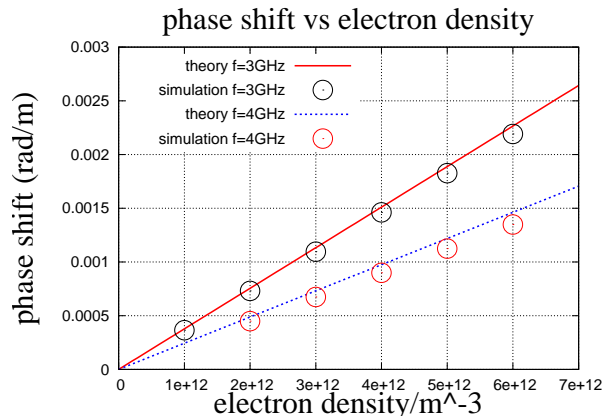


Figure 3: A comparison of phase shift obtained from computation and analytically

Figure 3 shows the theoretical and computed phase shifts for two values of frequencies. It may be noted that the approximations made in deriving the analytic result become more valid with decreasing electron densities. Most values of electron densities used in the computation are well

above those occurring in real machines. Despite this, the computational results compare very well with the analytic ones and they also show that the two results get closer with decreased electron density. The results are shown for two values of microwave frequencies and the cutoff frequency was 2GHz for both cases.

## A BRIEF NOTE ON EXPERIMENTAL MEASUREMENT OF PHASE SHIFT

The phase shift can be measured from the output frequency spectrum of the microwave. Due to the presence of a gap in the bunch train, the electron cloud is periodically cleared. This results in a phase modulation of the microwave at the frequency of the clearing of the electron cloud. In the presence of a single gap, this frequency would be the revolution frequency. Assuming that the modulation is sinusoidal and that the electron cloud gets completely cleared in the presence of the gap, we get for the wave amplitude,

$$\begin{aligned} w(t) &= \cos[\omega t + \Delta\Phi \cos(\omega_m t)] \\ &= \cos(\omega t) \cos[\Delta\Phi \cos(\omega_m t)] \\ &\quad - \sin(\omega t) \sin[\Delta\Phi \sin(\omega_m t)]. \end{aligned} \quad (5)$$

Using the expansion for  $\cos[\Delta\Phi \cos(\omega_m t)]$  and  $\sin[\Delta\Phi \sin(\omega_m t)]$ , we get

$$\begin{aligned} w(t) &= J_0(\Delta\Phi) \cos(\omega_m t) \\ &+ J_1(\Delta\Phi) \cos[(\omega + \omega_m)t + \frac{\pi}{2}] \\ &+ J_1(\Delta\Phi) \cos[(\omega - \omega_m)t + \frac{\pi}{2}] \\ &+ J_2(\Delta\Phi) \cos[(\omega + 2\omega_m)t + \pi] \\ &+ J_2(\Delta\Phi) \cos[(\omega - 2\omega_m)t + \pi] \dots \end{aligned} \quad (6)$$

Over here  $\omega_m$  is the modulation frequency and  $J_n$  are Bessel functions of the first kind. The amplitudes of the secondary peaks may be related to  $\Delta\phi$ . This technique suggested in Ref [1] has been employed in the experiments at PEP-II [4] and secondary peaks have been observed at the expected frequency.

## SUMMARY

Simulation results of the propagation of microwaves through electron clouds show good agreement with those obtained through analysis. A formulation that predicts the phase shift due to the electrons confined within the finite boundaries of a beam pipe has been presented. Calculations included a regime that exceeded realistic values of electron cloud densities, thus showing that real machine conditions fall well within the domain in which linear theory is valid. This is an update on work in progress and we intend to include more physical features into the simulation and the analytic models in the future. Notable among

them would be the effect of the periodic passage of a beam. Given the relativistic mass of the beam particles, it is clear that the beam would not contribute to the dispersion relation in the form of a plasma. However, the beam provides a time varying electric field and the extent of such an effect on the dispersion relation still needs to be estimated. The geometry of the cross section will become important in the presence of a beam and so we need to move to more realistic geometries.

The influence of external magnetic fields also needs to be investigated. In the presence of a solenoidal field, the microwave breaks into a left and right circularly polarized wave (known as L and R wave respectively). Both of these will have different dispersion relations. This will affect the outgoing wave in two ways. Due to the modified dispersion relations, the effective phase shift will be different. In addition to this, since the left and right circularly polarized waves undergo different phase shifts, there will be an overall rotation in the plane of polarization of the input microwave. One must also note that the presence of a solenoidal field results in a higher electron density near the walls, which would also influence the overall phase shift.

When a dipole magnetic field is present, the dispersion relation depends upon the direction of the wave electric field with respect to the external magnetic field. When they are parallel to each other, the wave does not see the magnetic field. This component is known as the ordinary wave. When the direction of the two fields are perpendicular to each other, it gives rise to the so called extraordinary wave. Since the dispersion relation of both these components are different, we will see a modified phase shift depending upon the strength of the magnetic field accompanied with a rotation of plane of polarization if applicable. Once again, the distribution of the electrons is not uniform in this case too, and this will have an independent effect upon the overall phase shift.

The PIC code VORPAL is well equipped to include all the above mentioned features into the simulation process. Understanding the effect of all these features and simulating their effects is important in order to correlate experimental observations with the properties of the electron distributions.

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